MEASUREMENT OF CAPACITY IN TRANSMISSION LINES AND CABLES

BY

R. M. HEIM
W. DUMKE

ARMOUR INSTITUTE OF TECHNOLOGY

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MEASUREMENT OF CAPACITY IN TRANSMISSION LINES

AND CABLES.

A THESIS

PRESENTED BY

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AND
WILLIAM DUMKE

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TABLE OF CONTENTS.

Title page---------- page 1
Preface---------- " 3
Introduction---------- " 4
Part One---------- " 5
Part Two---------- " 12
Preliminary apparatus-(sketch)------- " 14
Trans. line " (sketch)------- " 20a
Cables ---------- " 27-30
Three-phase measurements ---------- " 34-44
Data:-
" ,Preliminary ---------- " 18-19
" ,Single-phase ---------- " 23-24
Cables, data ---------- " 28-32
Three-phase, data ---------- " 35-43

24877

TABLE OF CONTENTS.

Title Page----------Page 1
Preface----------" 3
Introduction----------" 4
Part One----------" 8
Part Two----------" 12
Profiteers (poetic)-------" 14
Trees, Time " (poetic)-------" 20
Cactus----------" 23-30
Corn-flour Manufacturers----------" 24-44
Tube:-
" Prelimineries----------" 45-49
" Bridge-Piece----------" 53-54
Letter, Date----------" 55-56
Three-Piece, Size----------" 57-62

Preface.

In this work we have endeavored to obtain experimental values for transmission line capacities without actually measuring the capacity in an experimental line.

We have tried to show that through an analogy between capacity and conductance a value for capacity under given conditions may be obtained by measuring conductance through a different medium under somewhat similiar conditions a constant factor to obtain the capacity desired.

So far as we have been able to find out, this experiment has never been carried out in a similiar manner.

Richard M. Heim.

William Dunke.

May 15th. 1914.

Preface.

Introduction.

We will first explain the theoretical analogy between capacity and conductance and show why we were justified in spending our time in this work. We will then show how we have experimentally verified the theoretical relationship, and then have gone farther by using this relationship to find capacities where the conditions are such that their values are difficult to calculate. We will give numerical results, as found by measurements, in the various parts of the experiment.

Investigation.

Part One.

The simplest case of a condenser is that formed by two parallel plate conductors with an insulating material between them. The capacity of the condenser is easily found from the expression:-

$$C = KS/4\pi D \times 9 \times 10^5 \text{ microfarads},$$

where K is the dielectric constant of the dielectric between the plates, S the area of each plate, and D the distance between the plates, (dimensions given in centimeters). Combining the constants in this expression into one new constant we have for the capacity:-

$$C = K'S/D \text{ microfarads.}$$

This form shows that the capacity of the condenser varies directly as the area of the plates and inversly as the distance between them.

Bearing the last statement in mind we will now assume two conducting plates with a conducting medium between them, the medium having uniform cross-section from one plate

Part Two.

to the other. This condition could be had by placing plates on opposite sides of a conducting cube or on the ends of a conducting cylinder. The resistance of the block of material can be calculated from the formula:- $R = rD/S$ where S is the area of the cross-section of the medium in square centimeters, D is the distance between plates in centimeters, and r is the resistance of a cubic centimeter of the conducting medium.

This formula shows that the resistance varies directly as the distance between faces and inversely as the area of cross-section, a well known fact. This relation is just the reverse of the relation between the capacity and D and S as was shown on the previous page. The conductance of the conducting medium, however, is the reciprocal of the resistance and it must therefore bear the same relation to D and S as the capacity, except that a different constant factor is involved which depends upon the material making up the medium.

Bearing this in mind, we would expect that for a condenser having the same dimensions as a conducting material the ratio of the capacity of the first to the conductance of the second would be the same for any size if the same materials were used. Therefore, if this ratio were once determined and it were desired to find the capacity of a condenser without measuring it, the value could be found by measuring the conductance of the conducting material of the same size and shape as the dielectric in the condenser, and multiplying the measured value by the ratio found.

All this when the so-called electrostatic lines or the current lines are straight and are parallel to each other. Our problem was first to determine whether the analogy would hold in the case of diverging curved or straight fields, crowded and contorted fields, etc.

In small increments of volume where there is a contorted field the lines may be assumed

to be parallel,and in a conducting medium the resistance of the whole would be the summation of all of the separate small resistances. In a like manner the capacity of a condenser having a contorted field could be found. It was our first desire to show that the constant relation between conductance and capacity,which has been explained above,would hold true when the electrostatic and current fields had identical shapes.

Let us first take up the case of a single conductor surrounded by an insulating material and a lead sheath. The elctrostatic capacity between the conductor and the lead sheath is found theoretically as follows:-

Assume a ring of insulation of thickness dr, of radius r, and of unit length. The path of the flux is from the inner surface to the outer surface and the area of cross-section is $2\pi r$. If the whole path of the flux from the conductor to the sheath is cut into successive rings of thickness dr each ring may

be taken as a condenser having a capacity of K x 2πr/dr, where K is the dielectric constant of the insulating material. The capacity of the unit length of cable may be found from equation: $1/C = 1/c + 1/c' + 1/c'' + \text{-----} + 1/c^n$, where c,c',c" --- etc. are the capacities of the successive rings. Substituting the values of the successive capacities we have for the reciprocal of the total capacity the following:-

$$1/C = \int_{r'}^{r''} \frac{dr}{K \times 2\pi r} = \frac{\log r''/r'}{K \times 2\pi},$$

where r" is the inner radus of the sheath, and r' is the radius of the conductor. Therefore, the expression for the capacity, after combining the constant factors is:-

$$C = \frac{K'}{\log r''/r'} \text{ microfarads per unit length.}$$

For any length of cable L, the capacity will be LC.

If the insulating material were replaced by a conducting material the resistance would be found in a similiar manner. The resistance of a ring of thickness dr is R' dr/2πr, where R' is the specific resistance of the material.

The total resistance would be the sum of the individual resistances, or :-

$$R = \int_{r'}^{r''} \frac{R'dr}{2\pi r} = \frac{R'\log_e r''/r'}{2\pi} \text{ ohms per}$$

unit length. Combining the constant quantities into a single constant, K", and allowing for any length of cable L, the equation becomes :-

$$R = \frac{K''\log_e r''/r'}{L} \text{ ohms.}$$

The conductance is the reciprocal of this expression, or, denoting it by H, :-

$$H = \frac{L}{K''\log r''/r'} = \frac{1K''}{\log r''/r'} \text{ mhos where}$$

K"' is equal to 1/K".

This last expression is similiar to the capacity formula given above and differs from it only by a constant factor which is the same as that found in the parallel plate formula.

With this knowledge on hand we set about to determine experimentally whether this relation was constant for various forms of capacities and conductances. Our aim has been directed principally toward cables and aerial transmission lines, the

The usual reciprocal would be the use of the
logarithmic relationship, or :-

$$\beta = \frac{\log_e \frac{p_1}{p_2}}{v_2 - v_1} \text{ ohms per}$$

unit length. Combining the measured resistance into a single constant, β', and altering the unit of length to $\frac{1}{n}$, the equation becomes :- $\beta = \frac{\log_e \frac{p_1}{p_2}}{v_2 - v_1}$ ohms. The constant, or, treating it as $\frac{\beta'}{n}$, :-

$$\beta = \frac{n \log_e \frac{p_1}{p_2}}{v_2 - v_1} \text{ ohms where}$$

$\frac{p_1}{p_2}$ is equal to $\frac{1}{C}$.

This last expression is obtained in the capacity formula given above when used differentiating plate charge.

Plus this knowledge we could now get about in darkness completely ignorant of where in the room we were, but then living organisms, in general, have developed sensitivities and acutely investigative tissue, its

method of procedure being given in the next part of our thesis.

method of procedure being given in the next
part of our theorem.

Part two.

The apparatus which we used was simple, original, and home-made. We obtained a piece of tinned sheet iron about 20 by 28 inches and drilled several holes near the center of the sheet. Three of these holes were on the corners of a right angle triangle, two of whose sides were equal and were two inches from center to center of the holes. The holes were .02 inch in diameter. Through these holes we inserted bare No. 24 copper wire which we had previously tinned with solder. This wire has the same diameter as the holes, and the solder did not alter the size materially. We soldered the wires into the holes by passing a heavy current through the wire from one side of the plate to the other and thus heating it enough to melt the solder and make a good connection. If a fillet was formed on either side of the plate we cut it away with a knife so that the only connection between the wire and the plate was on the inner surface of the hole in the

Part two.

The apparatus which we used was simple, original, and home-made. We obtained a piece of tinned sheet iron about 6½ by 10½ inches and drilled several holes near the center of the sheet. Three of these holes were on the corners of a right angle triangle, two of whose sides were equal and were two inches from one another to center of the holes. The holes were .02 inch in diameter. Through these holes we inserted bare No. 24 copper wire which we had previously tinned with soldier. This wire has the same diameter as the holes, and the solder did not alter the wire materially. We soldered the wires into the holes by passing a heavy current through the wire from one side of the plate to the other and then heating it enough to melt the solder and make a good connection. If a filler and funnel is either side of the plate we cut it away with a knife so that the only connection between the wire and the plate was on the inner surface of the hole in the

FIG. 2.

FIG. 1.

plate. The arrangement of the wires in the plate is shown in fig.1.

It will be readily seen that if current is passed from one wire to the other the path will not be a straight line from one wire to the other, but it will take a form of a magnetic field between a north and a south pole, or an electrostatic field between opposite charges. This form of field is brought about by the repulsion bewteen like particles as they travel from the one point to the other. If this be the form of the current field we would expect the conductance from one wire to the other to vary by the same law with which the capacity between parallel wires varies.

The expression for the capacity between two wires in air is as follows:-

$$C = \frac{K}{\log d/r},$$

In which K is a constant containing several other quantities, d is the distance between centers of the wires, and r is the radius of the wires.

plate. The arrangement of the wires in the plate is shown in fig.1.

It will be readily seen that if current is passed from one wire to the other the potential difference between a point and a point midway between the wires will be zero, or an electrostatic field between adjacent wires. This form of field is brought about by the regularly spaced like particles as they travel from the one point to the other. If this be the form of the current field we would expect the conductance from one wire to the other to vary by the same law with which the capacity between parallel wires varies.

The expression for the capacity between the wires is as follows:-

$$C = \frac{E}{\log d/r},$$

In which E is a constant depending between other conditions, d is the distance between of the wires, and r is the radius of the wires.

The proof of this can be found in any text book dealing with electrostatics, and we will not go into it now.

If the conductance between the wires in the sheet varies inversely as the logarithm of the ratio of d to r, then there will be a constant relation between the conductance and the capacity for various ratios of d to r. Also, if the conductance varies in this manner the resistance between wires would necessarily have to vary directly as the logarithm of the ratio of d to r. The truth of this was what we have set out to determine.

It was a very simple thing to measure the resistance from one wire to the other. We placed a storage battery, an ammeter, a carbon plate rheostat, and the metal sheet in series. The current passed into the sheet through one wire and out through another two inches away, as shown in fig. 2. The part of the wire extending out on the back side of the sheet carried no current as there was no connection

The proof of this can be found in any text
book dealing with electrostatics, and we
will not go into it now.

If the conductance between the plates in-
stead varies inversely as the logarithm
of the ratio of S to r, then there will be a
constant relation between the conductance and
the capacity for various ratios of A to r.
Also, if the conductance varied in this manner
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as shown in fig. 2. The part of the wire ex-
tending out on the back side of the sheet cur-
ried no current as there was no connection

beyond the sheet. Therefore the current in the line as shown by the ammeter was the current passing through the sheet.

We had soldered to the ends of the wires sticking out on the back of the sheet other small wires which we connected to a Leeds & Northrup potentiometer. When the current had been adjusted to a definite value we measured the drop between the wires in the sheet very accurately. From this and the current we determined the resistance from Ohm's law. We checked the readings by using various currents and measuring the drops across the wires.

It was necessary to determine whether or not the metal sheet was homogeneous, or, better, whether the resistance in one direction was the same as the resistance in another direction. We did this by shifting our circuit to the two wires on the side of the triangle which was perpendicular to the first. We measured the drops over these wires, using the same current values as first used. The

resistance found in this case was slightly different from the other although the spacing was the same. This was due either to a poor connection between one of the wires and the sheet, or to unequal directional resistance.

Using an average of the two measured resistances, we calculated what the resistance across the hypotenuse would be, using the law that the resistance varied as the log d/r. Then, to find if it did vary in this manner, we measured the resistance across the hypotenuse in the same manner as ascribed above. The measured value checked the calculated value very closely, showing that our assumption was probably correct. The results obtained are shown on the next page.

Desiring to get more accurate results, we obtained a sheet of a higher grade sheet iron whose thickness was only one-half that of the first sheet. We pictured a transmission line tower with its base at the bottom

Data.

Radius of wires - - - - - - - - - - - - - - .01"

Distance between centers of A & B - - - - 2"

" " " " B & C - - - - 2"

" " " " A & C - - - -2.83"

Log 200 = 2.301 Log 283 = 2.452

A B

Volts	Amps.	Ohms.
.00465	10	.000465
.00371	8	.000464
.00278	6	.000464
.00185	4	.000463
		Av.= .000464

B C

Volts	Amps.	Ohms.
.00466	10	.000466
.00374	8	.000467
.00281	6	.000469
.00188	4	.000468
		Av.= .000468

Table.

Data. (continued)

A C

Volts	Amps.	Ohms.
.00493	10	.000493
.00394	8	.000493
.00296	6	.000494
.00198	4	.000496
	Av.*	.000494

Av. of A B and B C resistances = .000466 ohms.

$$\frac{\text{Log } 283}{\text{Log } 200} = \frac{2.452}{2.301} = 1.065$$

1.065 x .000466 = .0004965 ohm.

of the sheet, and its top about two-thirds of the way up when the sheet was standing on its long edge. We designed this tower to carry two-three-phase lines and a ground-wire. The most common methods of transmitting three-phase power are by flat spaced conductors and by equilateral triangular spaced conductors, the latter being the most common of the two. We placed two sets of each system on the tower to be used separately. We spaced the holes in the sheet accurately and made all distances between wires in one line equal to two inches, that is, an equilateral triangle two inches on a side, or three in a straight line the outer wires being two inches each from the central wire. We soldered a No. 24 wire into each hole by tinning it and inserting it into the hole and passing a heavy current through it. We soldered a small wire to the short end of each wire for the purpose of measuring the drop for any particular measurement.

In order to test the quality of the solder-

of the sheet, and its top about two-thirds of the way up from the sheet and standing on its long edge. To designed this paper to carry two-three-phase lines and a ground-wire. The most common methods of transmitting three-phase power are by flat spaced conductors and by equilateral triangular spaced conductors, the latter being the most common of the two. We placed two sets of each system on the tower or to be used concurrently. By spacing the holes in the sheet accurately and made all distances between wires in one line equal to two inches, that is, an equilateral triangle two inches on a side, or three in a straight line the outer wires being two inches each from the central wire. We numbered a fig. 91 wire into each hole by starting it and inserting it into the hole and passing a heavy current through it. We softened a small wire in the short end of each wire for further measurement.

In order to test the quality of the weather-

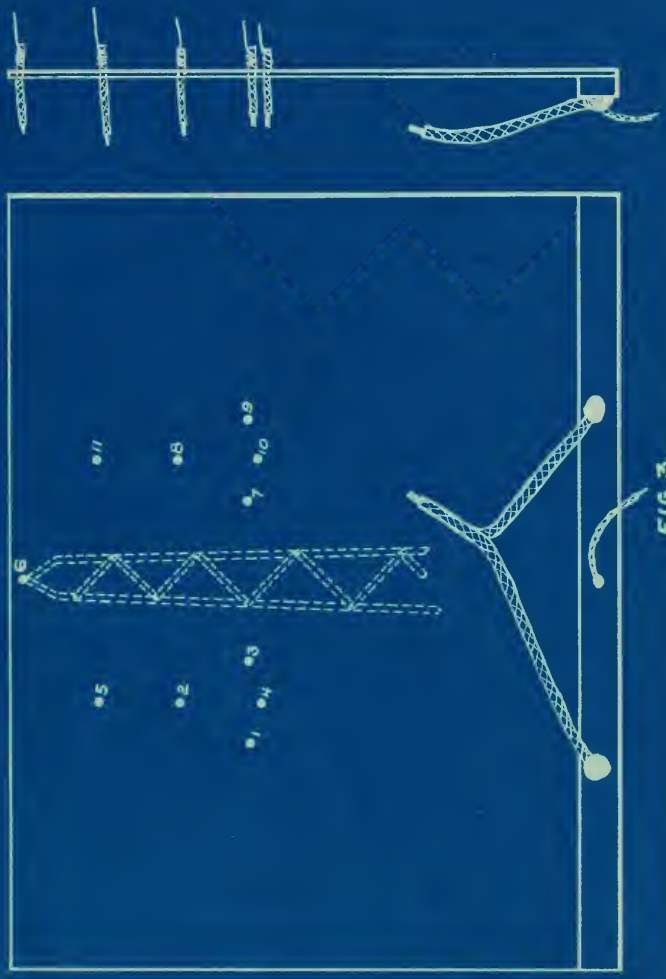

FIG. 3.

ed joint of each wire we measured the resistances between each pair that were two inches apart, and found them to check each other very well. There were ten of these pairs to be measured and one or two gave slightly higher resistances, showing a poor joint. By passing a heavy current through the defective wire the solder was again melted and a new joint was made. Generally one reheating process was sufficient to make a perfect joint and give the same resistance between wires as the other pairs. We found the directional resistance to be uniform throughout the sheet. We also measured some of the resistances between wider spaced wires and found them to vary exactly as the log d/r, when the value of the resiatance in the two-inch spacing was used as a basis of calculations. This proved that the conductance between the wires varied directly as the capacity between two parallel wires having the same ratio of d to r. Therefore there is a constant factor by which the conductance between wires of any

spacing may be multiplied to get the capacity of a single-phase line having the same value for d/r.

The capacity of a single-phase line is found by the expression:-

$$C = \frac{.03883}{\text{Log}_{10} d/r} \text{ microfarads per 1000 feet.}$$

This formula for capacity in a transmission line is calculated for line to neutral capacity, so that for line to line capacity the constant must be divided by two, as two equal capacities in series give one-half the value of either.

The following pages show the values which we have obtained experimentally for the resistances between wires separated by two and four inches. We have then shown how these values check our original assumption, and have determined the ratio between the capacity and conductance where the value of d/r is the same for both quantities. Having once determined this quantity we need but measure any conductance and use the factor

exponent may be multiplied to get the capacity of a single-phase line having the same value for d/r.

The capacity of a single-phase line is found by the expression:-

$$C = \frac{.0388}{\log_{10} d/r} \text{ microfarads per 1000 feet.}$$

This formula for capacity is a transformation like is employed for line to neutral capacity, so that for line to line capacity the constant must be divided by two, as the equal capacities in series give one-half the value of either.

The following gives some idea of the values we have calculated the ratio between the capacity and inductance for various values of d/r in the case for bi- cycles. Having measure my conductors and use the factor above determined with capacity to read off the values of d/r in the case for bi- cycles. Having

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Data.

Res.between	Volts	Amps.	Res.(ohms)
1 - 2	.00140	2	.000700
1 - 2	.00352	5	.000704
2 - 3	.00140	2	.000700
2 - 3	.00355	5	.000710
1 - 3	.00140	2	.000700
1 - 3	.00354	5	.000708
7 - 9	.00140	2	.000700
7 - 9	.00354	5	.000708
8 - 9	.00140	2	.000700
8 - 9	.00352	5	.000704
7 - 8	.00140	2	.000700
7 - 8	.00355	5	.000710
8 - 11	.00140	2	.000700
8 - 11	.00352	5	.000704
8 - 10	.00140	2	.000700
8 - 10	.00352	5	.000704
4 - 2	.00140	2	.000700
4 - 2	.00351	5	.000702
2 - 5	.00140	2	.000700
2 - 5	.00353	5	.000706

Data.

Pipe.	Section.	Volts.	Amps.	Time. (secs.)
	1 - 2	.00120	2	.000120
	2 - 4	.00222	3	.000222
	3 - 5	.00240	12	.000240
	4 - 6	.00222	4	.000222
	5 - 7	.00120	2	.000120
	6 - 8	.00222	3	.000222
	7 - 9	.00120	2	.000120
	8 - 10	.00222	3	.000222
	9 - 11	.00120	2	.000120
	10 - 12	.00222	3	.000222
	11 - 13	.00120	2	.000120
	12 - 14	.00222	3	.000222
	13 - 15	.00120	2	.000120
	14 - 16	.00222	3	.000222
	15 - 17	.00120	2	.000120
	16 - 18	.00222	3	.000222

Data.(continued)

Res. between	Volts	Amps.	Res.(ohms)
10 - 11	.00158	2	.000790
10 - 11	.00399	5	.000798
4 - 5	.00159	2	.000794
4 - 5	.00401	5	.000802

Average Res. for 4" spacing ----- .000796 ohm

" " " 2" " ----- .000703 "

Log 200 = 2.301 Log 400 = 2.602

$$\frac{\text{Log } 400}{\text{Log } 200} \times .000703 = \frac{2.602}{2.301} \times .000703 = .000795$$

Capacity of line having d/r = 200:-

$$C = \frac{.03883}{\text{Log } d/r} = \frac{.03883}{2.301} = .01686 \text{ MF. per mi.}$$

This capacity is given from line to neutral plane.

Line to line capacity = .01686/2 = .00843 MF.per mile.

Conductance in 2" spacing equals 1/.000703 = 1423 mhos.

Ratio of capacity to conductance =

$$\frac{.00843}{1423} = .00000593$$

Therefore, .00000593 x Conductance = Capacity.

Para.(Continued)

Yea. Interest	Write	Appt.	Yea. Income
10 - 11	.00000	8	.00000
12 - 13	.00000	9	.00000
4 - 5	.00000	3	.00000
6 - 8	.00000	5	.00000

Average Yea. For 1" Equality ----- .00000 abs

" " " 1" " ----- .00000 "

$$\text{Yal Ann} = \frac{\text{Yal Ann}}{\text{Yal Ann}} \quad \text{Yal Ann} = \text{.00000} \quad \frac{\text{3.000}}{\text{3.333}} = \text{.00000}$$

Equality at Time Having E/λ = 000:-

$$G = \frac{\text{Yea } E/\lambda}{\text{Yea } E/\lambda} = \frac{\text{.00000}}{\text{3.333}} = \text{.00000 at.per yr.}$$

Time to Time Capacity = .11/E/\lambda = .00000 at.per

Hour.

Conductance in 1" Equality equals 1/.00000 =

1000 ohms.

Ratio of Capacity to Temperature =

$$\frac{\text{.0012}}{\text{1000}} = \text{.0000000}$$

Temperature .0000000 @ Conductance = Capacity.

to find the capacity wished.

In order to check the value of this ratio which we have found, we made a new determination of the value with different apparatus. To get the same value we necessarily had to use the same material for measuring conductance. We obtained the conditions found in a cable with a single conductor in the center by cutting a corner from the large metal sheet and soldering the piece over the opening in a heavy copper ring. The opening was machined true, and the diameter was 2.05". By heating the ring above the melting point of the solder, and making a thin liquid sheet of solder around one face of the ring, we were enabled to make a perfect joint between the sheet and the ring by clamping them together and allowing them to cool. We then soldered a copper rod to the sheet end on in the center of the ring. The rod was .25" in diameter. It had been squared off on the end so as to give contact at

every point on its circumference. We cut away any surplus solder which formed a small fillet around the rod, and soldered wires to the rod and ring to lead in current (see fig.4).

We passed current from the rod to the ring and measured the drop from one to the other. The ring, being of very heavy cross-section, was equipotential throughout for our purpose. We used several values of current and measured the drop by the potentiometer. From the values used we found the resistance of the medium between the rod and the ring, and from that, the conductance. The results of this test are shown on page 28. The capacity of the cable with a single conductor in the center is found from the expression:-

$$C = \frac{.03883K}{\text{Log } r''/r'} \text{ microfarads per mile,}$$

where K is the dielectric constant of the insulating material, and r" is the inner radius of the sheath, and r' is the radius of the conductor. To have the same conditions as in the transmission line, we assumed the dielec-

the transmission line, we assumed its dielectric medium to be homogeneous. To take the same conditions as in the condenser. To make the dielectric material, and r' is the inner radius of the sheath, and r'' is the radius of the conducting material, and r'' is the outer radius

$$C = \frac{\text{[illegible]}}{\log \frac{r''}{r'}},$$

where K is the dielectric constant of the insulating material between the conductors.

Then from the consideration:-

with a simple condenser in the number is found above we found. The capacity of the cable are [illegible]. The reciprocal of this basic

used we found the reactances of the mutual inductance. Then the values the ring by the parameters/ure. Then the values to exert external reforce of current and measured was conditionally circumflexed for our purpose. The ring, being of very heavy cross-section, and measured the drop from one to the other.

We passed current from the rod by the ring and ring to pass to current (see Fig.4). around the rod, and soldered wires to the rod only sufficient solder points formed a small fillet every point on its circumference. We cut away

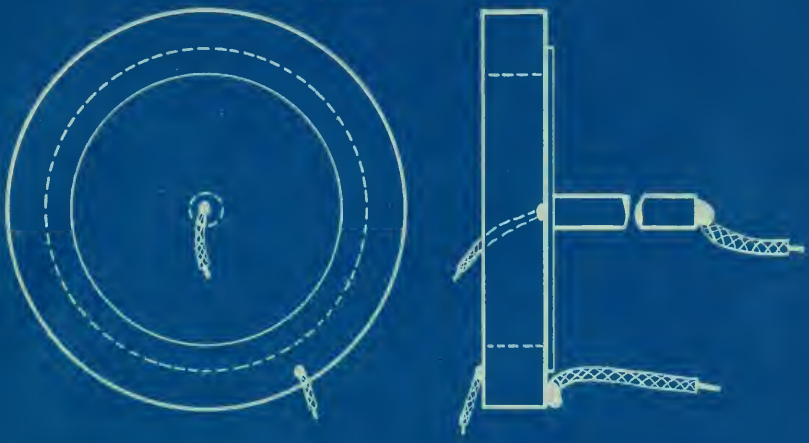

FIG. 4.

FIG. 5.

Data.

Amps.	Volts	Res.(ohms)
2	.00027	.000135
3	.00042	.000140
5	.00071	.000142
4	.00056	.000140

Average resistance ---------- .0001395 ohm.

" conductance-- 1/.0001395 = 7170 mhos.

Inner radius of ring---------- 1.025"

Outer " " rod---------- .125"

r"/r' = 1.025/.125 = 8.2 Log 8.2 = .914

$$C = \frac{.03883}{\text{Log } r''/r'} = \frac{.03883}{.914} = .0424 \text{ MF.per mile.}$$

Ratio of capacity to conductance =

$$\frac{.0424}{7170} = .00000592$$

Ratio as found on page 24 = .00000593

We will use the latter ratio as it was gotten by more reliable methods as an average of many separate measurements.

tric to be air, whose dielectric constant is unity. We determined the capacity and found the ratio between it and the conductance to be the same as the previously found ratio on page 24.

Feeling quite sure now that our assumptions were correct, we proceeded to measure other conductances where, under similar conditions the capacity is difficult to measure.

Our first test was that of a cable carrying two conductors centrally located and about one inch apart. We used the same ring and sheet as before, but removed the rod at the center and cleaned the surface where the rod had been soldered on. We then soldered two rods end on the sheet, one inch apart. This condition represents a single-phase cable with both lines insulated from the sheath and ground. By passing current from one rod to the other we were enabled to measure the resistance of the path between them. Having obtained the resistance be-

page 68.

FIG. 6.

tween the rods we multiplied its reciprocal by the constant ratio and found the capacity under similar conditions with air as a dielectric. For any other material as a dielectric the capacity would be multiplied by the dielectric constant of the material.

We then tried the cable carrying a single three-strand conductor in the center. We used three rods of the same size as used in the single conductor and two conductor cables, and soldered them to the center of the sheet after removing the other two (see fig.6). The three rods touched each other and were fastened together. We measured the resistance between the rods and the ring and calculated the corresponding capacity from the ratio found. The results of these two cable tests will be found on the next page.

On account of the smallness of these resistances last measured we did not attempt to work on a greater number of strands than three, nor did we attempt to try the three-phase cable

Data.

(1) Two conductors 1" apart in ring:-

Amps.	Volts	Res.(ohms)
2	.00048	.00024
3	.00072	.00024
4	.00096	.00024

Average resistance ---------- .00024 ohm

" conductance --- 1/.00024 = 4165 mhos

Capacity = .00000593 x 4165 = .0247 MF.per mi.

---

(2) Three-strand conductor in ring:-

Amps.	Volts	Res.(ohms)
5	.00056	.000112
6	.00067	.000112
8	.00090	.000113
10	.00115	.000115

Average resistance ---------- .000113 ohm

" conductance --- 1/.000113 = 8850 mhos

Capacity = .00000593 x 8850 = .0525 MF.per mi.

---

with single conductors on the corners of an equilateral triangle. The resistance in the three-phase cable would be close to 1/10,000 of an ohm, and, as the potentiometer reads only to 1/100,000 of a volt, an error of one point would make a considerable percentage error in the resiistance. If the material were very much thinner, and would therefore have a much higher unit resistance, or, if a higher resistance material were used, these measurements could be made with great accuracy.

Our next step was the three-phase transmission line measurement. As the capacity of a transmission line is affected by the presence of the earth below it, we represented a similar condition on our metal sheet by soldering a heavy copper bar across the bottom edge of the sheet. As the ground is merely a very good conductor in the immediate vicinity of a transmission line, this heavy rod answered our purpose very well.

We first measured the resistance from a single wire to the ground by passing current from a wire to the bar and measuring the drop from one to the other. In order to get various vertical distances above the ground, we used the three wires of each flat spaced line, one wire at a time, and measured the resistance to ground. This gave three pairs of wires, each pair having the same altitude. Taking the average of each pair we calculated the capacity between wire and ground where the ratio of altitude to radius of wire was the same. The results of this test will be found on the following page. These results show that the capacity from line to ground diminishes as the altitude increases, but not in the same ratio.

In the three-phase transmission line it is difficult to determine just what is meant by the capacity of the line, because there are three circuits involved. The capacity of one wire with respect to the other two can be found. This is the capacity which we desired

Data.

D/r	Amps.	Volts	Res.(ohms)
1200	2	.00120	.00060
"	4	.00240	.00060
"	2	.00120	.00060
"	4	.00240	.00060

Average resistance ---------- .00060 ohm

" conductance ---- 1/.00060 = 1670 mhos

Capacity = .00000593 x 1670 = .0099 MF.per mi.

D/r	Amps.	Volts	Res.(ohms)
1400	2	.00126	.00063
"	3	.00189	.00063
"	2	.00128	.00064

Average resistance---------- .000633 ohm

" conductance --- 1/.000633 = 1580 mhos

Capacity = .00000593 x 1580 = .00937 MF.per mi.

D/r	Amps.	Volts	Res.(ohms)
1600	2	.00134	.00067
"	3	.00201	.00067
"	2	.00136	.00068

Average resistance ---------- .000673 ohm

" conductance --- 1/.000673 = 1486 mhos

Capacity = .00000593 x 1486 = .00880 MF.per mi.

to find, because in an actual transmission line the charging current flows from two of the wires toward the third, due to one wire being always at a lower potential than the other two, or else two being at the same low potential, in which case all of the current passes from the one higher potential wire to the two lower. This can be easily seen from the potential curves in fig.7. Each curve represents the potential of one of the wires above or below neutral, plotted against time. The potential difference in any line at any instant is found by the distance between the curves representing the two wires in view. As the capacity is a constant quantity and cannot vary for a given line, except due to atmospheric conditions, we decided to use the points on the curves where we could most easily bring about the desired potential differences. For instance any easy condition is to have two wires at the same potential and the third to have the same difference with either.

in time, because in no normal transformation
Page
time the electric moments (Chapter 2) are the
wires toward the right, for he now sits be-
tic charge of a lower potential than the
other two, or else the latter at the same low
potential, in which case all of the expressed
because from the now higher potential wire to
the low lower. This may be modify seen from
the potential sources in Fig.7. Each source
represents the potential at one of the wires
wires at twice another, plotted against time.
The potential difference is only like as any
current to flow in the distance between the
be the capacity is a constant quantity and
current exponentially the rate wires to vary.
Instead very far a given time, would the to
attainable variations, we decided to use the
picture as the source which we could vary con-
trib varied above the desired potential differ-
ence. But inductance and capacitance is it
true the characteristic impedance of the

FIG. 7.

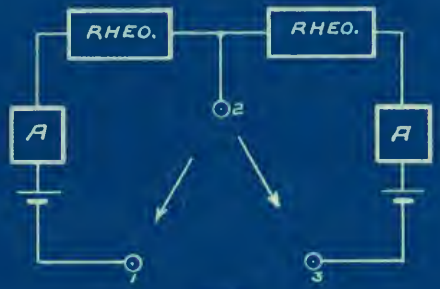

FIG. 8.

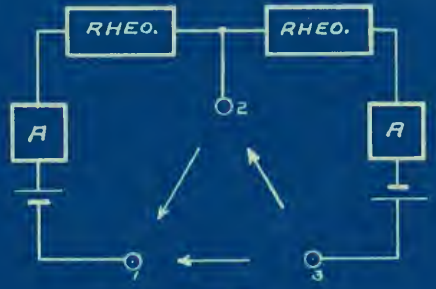

FIG. 9.

Another condition easily obtained is to have the same potential difference in two lines and twice this difference in the third line. This point on the curves is found at any time when the potential of any wire passes through zero.

We obtained the above conditions by making a complete circuit around through each line with a storage battery, a rheostat, and an ammeter in each circuit. The ammeters had all been calibrated by a standard. By varying the rheostats the polarities of the batteries we could bring about any instantaneous condition of potential as found from the three curves. By taking the battery out of one circuit and making the other two circuits carry the same currents, we were able to make the drop in the two circuits equal, and the drop in the third, which had no battery in it, either zero or double according as the direction of the current in one of the circuits. These two conditions are shown in

fig. 8 and fig. 9. The directions of the currents in the sheet are marked with arrows. We measured the resistance from one wire to the other two where the current travelled away from the one to the others, that is, in fig. 8, from 2 to 1 and 3, and in fig. 9, from 3 to 1 and 2. The current in each case was the sum of the two ammeter readings. The potential drop was the average of the two drops from the one wire to the two others. In fig. 8 the two drops were the same, but in fig. 9 one drop was twice the other, and the average was three-fourths of the greater. We made several determinations of resistance by both methods and took an average of the values found. These results will be found on the following page. There is a relation between the capacity of one wire with respect to the other two, and the capacity of one wire with respect to one of the others. This relation is very simple, and is stated, but not proved, in D.C. & J.P. Jackson, "AC and AC Machinery"

fig. 6 and fig. 8. The directions of the currents in the wheel are marked with arrows. To measure the resistance from one wire to the other use wheels 3 and 7, and in fig. 8, from 2 to 7 and 3, and in fig. 6, from 2 to 1 and 8. The current in each case goes the way of the low scale reading. The galvanometer used may be like sketch 42 or the type from the one wire to the other. In fig. 8 the two dynos were the same, but in fig. 6 one dyno was twice the other, if the intensity was three-fourths of the greater. We made several determinations of resistance by both methods and took an average of the values found. These results will be found on the following page. There is a relation between the capacity of one wire with respect to the other, and the capacity of any wire with respect to one of the others. This relation is very simple, and is stated, but not proved, in T.C. & J.P. Jackson, "AC and AC Machinery"

Data.

Drop between	Volts	Amps.
1 - 3	.005	4.75
1 - 2	.005	4.75

Total current = 9.5 Resistance =.000526 ohm

1 - 3	.005	4.75
2 - 3	.005	4.75

Total current = 9.5 Resistance =.000526 ohm

1 - 3	.0007	2
2 - 3	.0014	--

Average volts = .00105 Res. = .000525 ohm

1 - 2	.00140	4
1 - 3	.00281	--

Average volts = .00210 Res. = .000525 ohm

Average resistance ---------- .0005255 ohm

" conductance -- 1/.0005255 = 1904 mhos

Capacity of any wire with respect to other two,

C = .00000593 x 1904 = .01129 MF. per mile.

Data.

Log Interval	Volts	Amps.
1 - 2	.705	4.75
2 - 3	.705	4.75
1 - 3	.705	4.75
2 - 3	.705	4.75
1 - 2	.5897	3
2 - 3	.5915	--
1 - 2	.5914	4
1 - 2	.5897	--

page 931, as follows:- "Considering a three-phase circuit with the conductors at the corners of an equilateral triangle and balanced voltage, the electrostatic capacity of one of the conductors with respect to the other two is four-thirds the capacity with respect to only one of the others; but the voltage impressed on this capacity is the voltage reaction from the first conductor to the average or mid-voltage of the other two. " Testing this statement out with the results we had already obtained we found the following: (page 24)

Capacity from line to line equals .000843 MF.

" of three-phase system = .001129 "

4/3 x .000843 = .001125 MF. per mile.

We were not able to determine the reason for this relationship, but found it to be true to within one-third of one percent.

We next directed our attention to the three-phase line with flat spacing. We made circuits exactly like the circuits in fig.8

and fig.9, the only difference being in the fact that the wires 1, 2, and 3 were in a straight line. In this system the capacity of the central wire with respect to the other two is greater than that of an outer wire with respect to the others. In transposition along the line, each wire, however, assumes each of the three positions successively, and the total capacity will be the average of the three. We measured the resistances for each position in the same manner as before in the triangular spacing (page 39).

Theoretically the capacity of a three-phase line with triangular spacing is as follows:- $C = \frac{.03883}{\text{Log } D/r}$ MF. per mile from line to neutral. For flat spacing this value becomes:

$C' = \frac{.03883}{\text{Log } 1.26 D/r}$ MF. per mile.

D = distance between wires. r = radius of wires.

Log D/r = Log 200 = 2.301

Log 1.26 D/r = Log 252 = 2.4015

Ratio of C' to C = 2.3015/2.4015 = .9582

We calculated the capacity for flat spac-

and Fig.9, the only difference being in the
fact that the wires 1, 2, and 3 here is a
straight line. In this system the capacity
of the central wire will replace to the other
two is greater than that of no outer wire with
respect to the others. In consequence about
the line, each wire, however, because each of
the three positions successively, and the in-
ral capacity will be the average of the three.
We measured the resistances for each position
in the same manner as before in the triangular
spacing (page 80).

Theoretically the capacity of a three-
phase line with triangular spacing is as fol-
lows:- C = ---------- MF. per mile from line to
neutral. For first spacing this value becomes:
.meter
C' = ----------MF. per mile.
.meter
M = distance between wires. r = radius of wire

log D/r = log 200 = 5.301

log 1.25 D/r = log 250 = 5.3979

Ratio of C' to C = 5.3979/5.301 = .9824

We calculated the capacity for first spac-

Data.

Drop between	Volts	Amps.	Res.(ohms)
1 - 2	0	0	--
2 - 3	.00122	1	--
3 - 1	.00122	1	.000610
2 - 3	.00083	2	--
1 - 2	.00083	2	--
1 - 3	.00167	--	.000623

Average resistance by two methods=.000617 ohm

1 - 2	.00104	1	--
2 - 3	.00104	1	--
3 - 1	0	0	.000520
3 - 2	.00208	2	--
2 - 1	.00208	2	--
1 - 3	0	0	.000520

Average resistance = .000520 ohm.

" "n for finding capacity =

$$\frac{.000520 + .000520 + .000617}{2} = .000552 \text{ ohm}$$

Average conductance = 1/.000552 = 1812 mhos.

" capacity = 1812 x .00000593 = .01075 MF.

" " as found by theory =
.01129 x .9582 = .01082 MF. per mile.

These values check within .6%.

Table.

Prop between	Volts	Amps.	Res.(ohms)
1 - 2	0	0	--
2 - 3	.00320	1	--
3 - 1	.00320	1	.000013
2 - 3	.00320	2	--
1 - 2	.00320	2	--
1 - 3	.00320	--	.000013
1 - 2	.00320	1	--
2 - 3	.00320	1	--
3 - 1	0	0	.000013
2 - 3	.00320	2	--
3 - 1	.00320	2	--
1 - 2	0	0	.000013

Average resistance = .000013 ohm.

" " for floating magnet = .000013 ohm.

Average resistance = 1/.000039 = 1272 ohms.

" capacity = 1272 x .0000008 = .0173 mf.

" " in Farad by theory = .0173 mf. per mile.

These values agree well.

ing and tested this theoretical ratio and found it to hold true fairly closely. The results are shown on the preceeding page. These were our last measurements.

During the course of our experimenting which covered a period of several months, we made many observations which are not shown in this thesis. The most important relation and results are shown, but we did not formulate any laws regarding these relations nor did we make any determinations which were absolutely unknown or impossible to make by other means. We think, however, that we have shown an easier method to make measurements of capacities under some conditions. We hope that others will take up this experiment and carry it out to more detail and on a larger scale.

- END -

more detail but on a larger scale.

Take of this experiment may merely it out to be some conditions. We hope that others will method to make measurements of expansions under pressure, this we shall by other means. We think, however, that we shall by water methods or impossible to make by other means. may have required. Since relations our did we results are shown, but we did not determine this theory. The most important relation not made many observations which are not shown in which covered a period of several months, we their importance.

During the course of our experimenting it to hold true fairly closely. The results are shown on the preceding page. These were our ing and tested this theoretical rule and found